6.5 First of all write $p_0(\triangle t)$ as an exponential function:

$$p_0(\triangle t) = (1 - \lambda \triangle t)^{\frac{t}{\triangle t}} = \exp\left(\frac{t}{\triangle t}\ln(1 - \lambda \triangle t)\right). \tag{1}$$

Recall Taylor expansion of $\ln(x)$

$$\ln(x + \triangle x) - \ln(x) = \frac{\triangle x}{x} + o(\triangle x),$$

and use this result with $x = 1$ and $\triangle x = -\lambda \triangle t$:

$$\ln(1 - \lambda \triangle t) = -\lambda \triangle t + o(\triangle t). \tag{2}$$

Substitute (2) into (1):

$$p_0(\triangle t) = \exp\left(-\lambda t + \frac{o(\triangle t)}{\triangle t}\right).$$

By definition $\lim_{\triangle t \to 0} \frac{o(\triangle t)}{\triangle t} = 0$, moreover $\exp(x)$ is a continuous function of x, hence

$$\begin{aligned}\lim_{\triangle t \to 0} p_0(\triangle t) &= \lim_{\triangle t \to 0} \exp\left(-\lambda t + \frac{o(\triangle t)}{\triangle t}\right) \\ &= \exp\left(\lim_{\triangle t \to 0}\left(-\lambda t + \frac{o(\triangle t)}{\triangle t}\right)\right) \\ &= \exp(-\lambda t).\end{aligned}$$