11.9 To solve parts (a) and (b) proceed as in Exercise 11.8.
(a)

$$y(t,T)|\mathcal{F}_t \overset{Q}{\sim} N\left(r_t(T-t)+0.005(T-t)^2, 0.01\frac{(T-t)^3}{3}\right)$$

(b)

$$\begin{aligned} X_t &= X(t,r_t) = \mathrm{E}_t^Q\left[\mathrm{e}^{-y(t,T)}\right] \\ &= \exp\left(-r_t(T-t) - 0.005(T-t)^2 + 0.01\frac{(T-t)^3}{6}\right) \qquad (1) \end{aligned}$$

(c) Find the necessary partial derivatives:

$$\begin{aligned} \frac{\partial X}{\partial r_t} &= -(T-t)X, \qquad \frac{\partial^2 X}{\partial r_t^2} = (T-t)^2 X, \\ \frac{\partial X}{\partial t} &= \left(r_t + 0.01(T-t) - 0.005(T-t)^2\right) X. \end{aligned}$$

Plug this into the no-arbitrage Feynman-Kac PDE:

$$\begin{gathered} \frac{\partial X}{\partial t} + 0.01\frac{\partial X}{\partial r_t} + 0.005\frac{\partial^2 X}{\partial r_t^2} - r_t X_t \\ = \left(r_t + 0.01(T-t) - 0.005(T-t)^2\right) X_t \\ -0.01(T-t)X_t + 0.005(T-t)^2 X_t - r_t X_t = 0. \end{gathered}$$

Note that X in equation (1) also meets the boudary condition $X_T = 1$.