

13.18 Expected squared hedging error is easiest obtained from the formula

$$\text{ESRE}_t(H_{t+\Delta t}) = \text{Var}_t(H_{t+\Delta t}) - \frac{(\text{Cov}_t(H_{t+\Delta t}, X_{t+\Delta t}))^2}{\text{Var}_t(X_{t+\Delta t})}$$

which yields

$$\begin{aligned} \text{ESRE}_{l,t}(H_{t+\Delta t}) / \Delta t &= (H_{l+1,t} - H_{l,t})^2 \lambda_1 + (H_{l+2,t} - H_{l,t})^2 \lambda_2 \\ &\quad - \frac{\left(\sum_{j=1}^2 (H_{l+j,t} - H_{l,t})(e^{jJ} - 1)\lambda_j\right)^2}{\sum_{j=1}^2 (e^{jJ} - 1)^2 \lambda_j} + o(\Delta t), \end{aligned}$$

and after passing to the limit we have

$$\begin{aligned} G_l(t) &\equiv \lim_{\Delta t \rightarrow 0} \text{ESRE}_{l,t}(H_{t+\Delta t}) / \Delta t & (1) \\ &= \frac{\lambda_1 \lambda_2 (e^J - 1)^2}{\sum_{j=1}^2 (e^{jJ} - 1)^2 \lambda_j} \left((H_{l+2,t} - H_{l,t}) - (H_{l+1,t} - H_{l,t})(e^J + 1) \right)^2 & (2) \end{aligned}$$

The expression for the total hedging error reads

$$\begin{aligned} \varepsilon_{0L}^2 &= \int_0^T \text{E}_0[G(t)] dt \\ &= \int_0^T e^{-(\lambda_1 + \lambda_2)t} (G_0(t) + \lambda_1 t G_1(t)), \end{aligned}$$

since after a jump of size $2J$ the option is out of the money and the hedging error is therefore 0, i.e.

$$G_l(t) = 0 \text{ for } l \geq 2.$$

Substituting (1) and the results of Exercise 12.17 into the preceding formula we find

$$\begin{aligned} \varepsilon_{0L}^2 &= \frac{\lambda_1 \lambda_2 (e^J - 1)^2}{\sum_{j=1}^2 (e^{jJ} - 1)^2 \lambda_j} e^{-T(\lambda_1 + \lambda_2)} \int_0^T e^{-\eta(T-t)} \\ &\quad \times \left(\left(e^J H_{0,T} + (e^J + 1) H_{1,T} + e^J \lambda_1^Q H_{1,T}(T-t) \right)^2 + \lambda_1 t H_{1,T}^2 e^{2J} \right) dt \\ &= \frac{\lambda_1 \lambda_2 (e^J - 1)^2}{\sum_{j=1}^2 (e^{jJ} - 1)^2 \lambda_j} e^{-T(\lambda_1 + \lambda_2)} \\ &\quad \times \left(I(-\eta, e^J H_{0,T} + (e^J + 1) H_{1,T}, e^J \lambda_1^Q H_{1,T}) \right. \\ &\quad \left. + \lambda_1 H_{1,T}^2 e^{2J} \eta^{-2} ((\eta T - 1) + e^{-T\eta}) \right), \end{aligned}$$

where

$$\begin{aligned} \eta &= 2(\lambda_1^Q + \lambda_2^Q) - \lambda_1^P - \lambda_2^P, \\ I(\kappa, C_1, C_2) &= \int_0^T e^{\kappa(T-t)} (C_1 + C_2(T-t))^2 dt \\ &= \frac{-(C_1 \kappa - C_2)^2 + \kappa^2 e^{\kappa T} (C_1 + C_2 T)^2 + 2e^{\kappa T} C_2 (C_2 - C_1 \kappa - C_2 \kappa T) - C_2^2}{\kappa^3}. \end{aligned}$$

It is easy to verify that the hedging error is strictly positive, even though we are hedging continuously.