

1.14 a) The product PQ is

$$\begin{bmatrix} \frac{11}{90} & \frac{41}{90} & \frac{19}{45} \\ \frac{11}{120} & \frac{71}{120} & \frac{19}{60} \\ \frac{1}{5} & \frac{1}{5} & \frac{3}{5} \end{bmatrix}$$

and one can easily check that this is a probability matrix.

b) Let us define $\mathbf{1}$ to be a column vector of ones

$$\mathbf{1} = \begin{bmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{bmatrix}$$

Assuming that all the elements of P are positive, the fact that P is a probability matrix is equivalent to

$$P\mathbf{1} = \mathbf{1},$$

and the same holds for Q ,

$$Q\mathbf{1} = \mathbf{1}.$$

Now if P and Q have only positive elements then also PQ will have only positive elements. So we just need to verify that $(PQ)\mathbf{1} = \mathbf{1}$, but that is easy because matrix multiplication is associative (in a chain of multiplications the result is independent of the order in which intermediate multiplications are carried out),

$$(PQ)\mathbf{1} = P(Q\mathbf{1}) = P\mathbf{1} = \mathbf{1}.$$